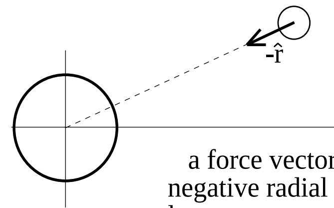
Chapter 10
Gravitational Chapter Review

EQUATIONS:

- $\boldsymbol{F} = G\dfrac{m_1 m_2}{r^2}(-\boldsymbol{r})$ [This is the technical presentation of the magnitude and direction of a gravitational force between masses m_1 and m_2, where r is the distance between each mass's center of mass and G is the Universal Gravitational Constant. Please note that the unit vector $-\boldsymbol{r}$ is a polar-spherical unit vector that denotes the fact that the force is always on line between the two bodies, and is always attractive. It is very rare that you will use this expression as a full blown vector. Most problems are done in a Cartesian coordinate system (not a polar spherical system), so the norm is to use this expression to determine the *magnitude* of the gravitational force, then assign a positive or negative sign depending upon how the force is oriented relative to the Cartesian coordinate axis being used in the problem. It has been presented in all of its glory because the notation is common in textbooks, but the $-\boldsymbol{r}$ notation is rarely explained.]



a force vector in the negative radial direction denotes an *attractive* force

- $|F| = G\left(\dfrac{m_1 m_e}{r_e^{\,3}}\right)r$ [The magnitude of the gravitational force on a mass m_1 when *inside* the earth (assumed homogeneous) at distance r units from the center of the earth. Note that as r_e and m_e are constants--the radius and mass of the earth, respectively--this force is a linear function of the position r. You should be able to derive this expression. That is, you should understand what was done in its derivation so that you could deal with an inhomogeneous earth.]

- $U = -G\dfrac{m_1 m_2}{r}$ [This defines the amount of gravitational potential energy that exists in a two-body system. You should be able to derive this. It comes from $U(r) - U(\infty) = -\int_{\infty}^{\boldsymbol{r}} \boldsymbol{F} \bullet d\boldsymbol{r}$, where $\boldsymbol{F}$ is the gravitational force $G\dfrac{m_1 m_2}{r^2}(-\boldsymbol{r})$ and the potential energy is defined where the force is zero (i.e., out at infinity). Note that the direction of the force is in the *negative radial direction*. If you don't like radial coordinates, change to *x's* and *dx's*, do the Calculus, then substitute the *r's* back into the expression.]

- $E_{tot} = -G\dfrac{Mm_1}{2r}$ [This is the total mechanical energy--*potential* plus *kinetic*--wrapped up in the motion of a single mass m_1 orbiting a considerably more massive object M in a CIRCULAR ORBIT of radius r. Note that this expression makes no sense if the motion is elliptical.]

- $v = \frac{2\pi r}{T}$ [This is the relationship between the magnitude of the velocity of a body moving in *circular orbit* of radius r and the period T of the motion, the period being the amount of time required to complete one full orbit.]

- $\frac{v^2}{r}$ [In orbital motion, this is the magnitude of the centripetal acceleration an object moving in a circular orbit must experience due to its gravitational interaction with the other body in the system. Note that r is the *radius* of the circular orbit.]

- mvr [This is the angular momentum ***rxp*** of a mass moving in a CIRCULAR orbit about a much larger mass (a satellite about a planet, for instance). Also, this is the angular momentum of a small mass moving in an elliptical orbit about a large body, where r is the distance between the small mass and the system's center of the mass (usually inside the large body). Note also that v must be the *component* of velocity vector that is *perpendicular* to ***r***. The angular momentum magnitude can additionally be written as $I\omega$ AS LONG AS ω is defined as $(v_{perpendicular})/r$ and I is the moment of inertia of a point mass, or mr^2.]

- $T^2 \alpha\, r^3$ [This is Kepler's Third Law. It states that the square of the period of a planet orbiting the sun is proportional to the cube of the orbit's semi-major axis. This is the *only one* of Kepler's laws that is an approximation. It isn't true if the masses involved are anywhere near equal in size.]

<u>COMMENTS, HINTS, and THINGS to be aware of</u>:

- This is a nice review chapter because it incorporates most of the concepts you have studied to date into the analysis of one *kind* of system--a gravitational one.

- **Angular momentum** in orbital motion is **conserved** whether the path is circular or elliptical. Stated mathematically, that relationship is $L_1 = L_2$.

- The **total mechanical energy** in orbital motion is **conserved** whether the path is circular or elliptical. Stated mathematically, that relationship is $E_1 = E_2$.

- For all motion in space in which **a** very **small body orbits a** very **large body**:
  --As there is little frictional drag in space, the total mechanical energy is conserved. If the motion is circular, that energy is $E_{tot} = -G\frac{Mm_1}{2r}$. If the motion is elliptical, the total mechanical energy must be written as the *potential* plus *kinetic energy* in the system, relative to the system's center of mass.
  --As there are no external torques acting (the gravitational force is along the line between the two bodies), angular momentum is conserved.
  --Kepler's Third Law holds in this situation ($T^2 \alpha\, r^3$).

- For motion in space in which **two similar sized bodies orbit** one another:
  --The two bodies orbit about the system's center of mass.

--The total mechanical energy and the angular momentum are conserved, relative to the system's center of mass.

- The fact that the **gravitational potential energy** function is negative shouldn't put you off. The only time you will ever deal with a potential energy function is when you are trying to determine a work quantity. Remember, work is related to potential energy *differences*.

- **Conservation of energy**, **conservation of angular momentum**, and **N.S.L.** coupled with **centripetal acceleration** are the main stays of analysis when dealing with gravitational systems. There will be times when you will be asked to determine quantities that seem to be completely off the wall--the *period* of an orbiting body's motion, for instance. When in doubt, write down every expression you can think of that is true for the system, then go from there.

- An easy way to remember **Kepler's Three Laws** is to remember their names:
  --The Law of Orbits: Planets move in *elliptical orbits* having the *sun at one focal point*.
  --The Law of Areas: A line joining any planet to the sun sweeps out *equal areas in equal times* (i.e., $dA/dt = constant$).
  --The Law of Periods: The *square of the period* of any planet about the sun is proportional to the *cube of the planet's mean distance* from the sun.

- There are two **basic problem-types** to look for when dealing with gravity. The first is **orbital motion**. The second is **free fall**. Be sure you know which you are dealing with. For instance, the total mechanical energy in a free fall problem is NOT $E_{tot} = -G\frac{Mm_1}{2r}$ (that expression relates total mechanical energy in a circular orbit situation where one body is a lot larger than the other). Know which relationships are relevant, and when!